FMP-I
Forward and Futures
Introduction - Derivatives

- Derivatives are financial instruments which can be traded in the market.
- Derivatives derive their value from an **underlying asset** and some other variables such as interest rates, volatilities etc.
- Futures, forwards, options and swaps are some of the most common examples of derivatives.
- The **underlying asset**: It is a more basic financial instrument. Example: stocks.

- Example of a derivative: Option:
  An investor owns a call option (which is a derivative) whose underlying asset is the common stock of a company A. This option gives the investor, the right to buy the stock at a certain predefined price on or before a future date.
Introduction - Markets

• Exchange traded Markets
  – Market where individuals trade standardized contracts that have been defined by the exchange themselves.
  – An Exchange acts as an intermediary which facilitates a regulatory oversight and hence makes the markets a safer place for trading
  – Chicago Board of Trade and Chicago Mercantile Exchange are two examples
  – Open outcry system and Electronic trading

• Over the counter markets
  – There is no intermediary and no standardized contracts, parties can be created their own T&C with each other.
  – Much larger than the exchange traded market in terms of value of underlying assets (more than 4 times larger)
  – Trades done between financial institutions or between financial institutions and clients. Financial institutions act as a market maker (quote both bid and offer)

Source of Graph: The Economist
Types of Investments and Purposes

• Asset Types
  – Financial Assets: Equity, Debt securities
  – Commodities: Gold, Copper, Crude Oil
  – Real Estate

• Let’s take an example of a financial asset (stock).
  – We can buy the stock through the broker by paying the stock price.
  – We can either hold the bought asset or sell it at the current market price.
  – During the holding period of the stock, the dividends received goes to your pocket as the income from the asset.
  – After selling the asset, we earn a profit or loss on the asset, depending on the selling price of the asset (stock).

• Purpose of Assets
  – Investment Asset
  – Consumption Asset

• Market Maker
  – An individual or an institution which keeps an inventory of financial instruments or commodities who could be asked for the trade of those assets. The individual or the institution then quotes a bid and an offer price on the option.
Consumption vs Investment Assets

• Investment assets
  – Assets held for investment purposes by significant numbers of investors. (examples: stocks, bonds, gold, silver)

• Consumption assets:
  – Assets held primarily for consumption (examples: copper, oil and pork bellies)

• Gambling – Short Selling an example
  – Short selling involves selling securities that are not owned.
    • Suppose an investor short sells 500 IBM shares, the broker will borrow the securities from another client and sells them in the market in the usual way.
    • At some stage the investor will close the position by purchasing 500 IBM shares. The investor takes the profit if the stock prices have declined, else vice versa.
    • Short Squeezed: If anytime the broker runs out of shares to borrow, the investor is short squeezed and forced to close his position immediately
Forward and Futures Contracts

- **Futures Contracts**: Agreement to buy or sell an asset for a certain price at a certain time. A futures contract is traded on an exchange.

- **Forward Contracts**: Forward contracts are similar to futures except that they trade in the over-the-counter market.

- **Notation for Valuing Futures and Forward Contracts**
  - $S_0$: Spot price of the asset underlying today
  - $F_0$: Futures or forward price today
  - $T$: Time until delivery date (in years)
  - $R$: Risk-free interest rate per annum, expressed in continuous compounding, for maturity $T$

- Payoff of forwards and futures:
  - **Long**: \[ P \times X \]
  - **Short**: \[ P \times S \]

In both Forward and Futures contracts there is an obligation to buy or sell an asset.
Notation for Valuing Futures and Forward Contracts

- Consider a stock price at $100 today and the borrowing rate is 8% for 1 year. What should the 1 year forward price of the stock be? When will you make a profit in this case?

\[ F_0 = S_0 e^{rT} \]
\[ F_0 = S_0 (1+r)^T \]

- Where:
  - \( S_0 \): Spot price today
  - \( F_0 \): Futures or forward price today
  - \( T \): Time until delivery date
  - \( r \): Risk-free interest rate for maturity \( T \)
Pricing and Valuation of Forward Contracts

- Arbitrage free Forward prices are given as:
  \[ F(0,T) = S_0 e^{rT} \]

- If we long the forward contract at time \( t=0 \), at forward’s price \( F(0,T) \), the initial cash outlay would be zero.

- At time \( t=t \), we have claim on the asset which is worth \( S_t \) and an obligation to pay \( F(0,T) \) at time \( t=T \).

- At time \( t=T \), we pay \( F(0,T) \) and receive the asset worth \( S_T \).

- Forward rates are rates of interest implied by the current zero rates for a period of time in the future.
Question: Forward Prices

We have the zero rates for year 4 and year 5 then the forward rate for the period of time between year 4 and year 5 would be known as the forward rate for that time period of 1 year.

\[
\begin{align*}
\text{Year 4} & \quad \text{Year 5} \\
F_4 &= 4\% & F_{4,5} & \quad F_5 = 5\%
\end{align*}
\]

Consider that you invest $100 for 4 years and then roll it forward for one year in the 5 year. Calculate the Forward rate \( F_{4,5} \)
Solution:

The total amount at the end of 5 years would be given as:

\[ 100 \times e^{0.04 \times 4} e^{F_{4,5} \times 1} \]

If the same $100 was invested for 5 years instead then it would grow to

\[ 100 \times e^{0.05 \times 5} \]

Equating the two with natural log we get forward rate \( F_{4,5} = 8.99\% \)

**Generalized formula for forward rate:**

- In above scenario, assume year 4 and 5 to be \( T_1, T_2 \) and zero rates as \( R_1, R_2 \), then

\[ e^{R_1 \times T_1} e^{F_{4,5} \times (T_2 - T_1)} = e^{R_2 \times T_2} \]

simplifying this we get a generalized formula for forward rate

\[ F_{4,5} (\text{forward rate}) = \frac{(R_2 T_2 - R_1 T_1)}{(T_2 - T_1)} \]

- Use this formula to solve the above question.
Options *(to be covered in detail in later slides)*

- Traded both on exchanges and over the counter markets
- Call option gives the holder the right to buy the underlying asset by a specified time at a certain price.
- Put option gives the holder a right to sell the underlying asset by a specified time at a certain price.
- European options can be exercised on the specified date only, unlike American options which can be exercised anytime up to the expiration date.
- One option contract is to buy/sell 100 shares in the US.