Fixed Income Securities - II
Agenda

- Reading 64: Introduction to the Valuation of Debt Securities
- Reading 65: Yield Measures, Spot Rates, and Forward Rates
- Reading 66: Introduction to Measurement of Interest Rate Risk (1hr)
  
  - Key Issues in Measurement of Interest Rate Risk
  - Price Volatility and Convexity
  - Effective Duration
  - Bond's Modified Duration
  - Alternative Definition of Duration
  - Duration of Portfolio
  - Convexity Measure of Bond
  - Modified and Effective Convexity
Key Issues in Introduction to the Measurement of Interest Rate Risk

- Measuring Interest Rate Risk
- Price Volatility
- Convexity
- Effective Duration
- Alternative definitions of Duration
- Duration of a portfolio
- Convexity measure of a bond
- Modified and Effective Convexity
- Price Value of a Basis Point (PVBP)
Measuring Interest Rate Risk

• Interest rate risk can be measured by two methods:
  – Full Valuation Method:
    • Under this method the normal valuation techniques are used to value a bond or a bond with embedded options.
    • When the interest rates change the entire exercise is repeated again to find out the value as per the new interest rates.
    • The two values are compared to arrive at the impact of change in interest rate.
  – Duration/Convexity Method: This gives an approximate result of the sensitivity of the bond. But it is much simpler compared to the full valuation method.
Price Volatility and Convexity

• We have already seen that the price-yield curve is a negatively sloped and is a curve.

• The curve of a Callable bond exhibits Negative Convexity. This is because the increase in the price of a security as a result of fall in the yield is capped at the call price. See the below graph:
Price Volatility and Convexity

- The curve of a Puttable bond exhibits Positive Convexity. This is because the decrease in the price of a security as a result of increase in the yield is limited to the put price. See the below graph:
Effective Duration

- Duration is the measure of how long on an average the holder of the bond has to wait before he receives his payments on the bond. A coupon paying bond’s duration would be lower than “n” as the holder gets some of his payments in the form of coupons before “n” years.

- Effective duration is calculated as:

\[
\text{Effective Duration} = \frac{\text{(Bond price when yield falls} - \text{Bond price when yield rises)}}{2 \cdot (\text{Initial Price}) \cdot (\text{Change in yield in decimals})}
\]

Percentage change in Bond Price = -Effective Duration * Change in yield in percent. (Δy)

- Consider a bond trading at 96.54 with duration of 4.5 years. In this case

\[
\Delta B = -96.54 \cdot 4.5 \cdot \Delta y
\]
\[
\Delta B = -434.43 \Delta y
\]

If there is 10 basis points increase (+Δy) in the yield then the bond price would change by:

\[
\Delta B = -434.43 \cdot (0.001)
\]
\[
\Delta B = -.43443
\]
Hence, B = 96.54 - .43443 = 96.10
## Bond modified duration calculation

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<th>Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
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<tbody>
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<td>Cash flows</td>
<td>5.125</td>
<td>5.125</td>
<td>5.125</td>
<td>5.125</td>
<td>5.125</td>
<td>5.125</td>
<td>5.125</td>
<td>105.1</td>
</tr>
<tr>
<td>$t^*F_t$</td>
<td>5.1</td>
<td>10.3</td>
<td>15.4</td>
<td>20.5</td>
<td>25.6</td>
<td>30.8</td>
<td>35.9</td>
<td>841.0</td>
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<tr>
<td>$(1+r)t+1$</td>
<td>1.1</td>
<td>1.2</td>
<td>1.3</td>
<td>1.4</td>
<td>1.4</td>
<td>1.5</td>
<td>1.6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.6</td>
<td>8.8</td>
<td>12.5</td>
<td>15.9</td>
<td>18.9</td>
<td>21.5</td>
<td>23.9</td>
<td>532.5</td>
</tr>
</tbody>
</table>

**Market rate**: 5.2%

- **Price**: 99.46
- **Modified duration**: 6.42
- **Duration**: 6.76
Alternative definitions of Duration

- **Macaulay Duration**: is the weighted average of the times when the payments are made. And the weights are a ratio of the coupon paid at time “t” to the present bond price.
- Macaulay duration is also used to measure how sensitive a bond or a bond portfolio's price is to changes in interest rates.

Where:
- \( t \) = Respective time period
- \( C \) = Periodic Coupon payments ; \( y \) = Periodic yield ; \( n \) = Total number of periods
- \( M \) = maturity Value

- Calculating Macaulay Duration:

\[
\text{MacaulayDuration} = \frac{\sum_{t=1}^{n} \frac{t \cdot C}{(1+y)^t}}{\text{Current Bond Price}} = \frac{n \cdot M}{\text{Current Bond Price}}
\]

\[
\begin{align*}
D &= \frac{40}{(1.05)}(1) + \frac{40}{(1.05)^2} (2) + \frac{40}{(1.05)^3} (3) + \frac{1040}{(1.05)^4} (4) \\
&= \frac{3636.76}{964.54} = 3.77
\end{align*}
\]

Note that this is 3.77 six-month periods, which is about 1.89 years.
Alternative definitions of Duration

- **Modified Duration**: is derived from Macaulay Duration. It is better than Macaulay Duration as it takes into account the current YTM.

\[
\text{Macaulay Duration} = \frac{\text{Macaulay Duration}}{1 + \frac{r}{n}}
\]

- **Effective Duration** calculations explicitly take into account the bond's option provisions. The other methods of calculation ignore the option provision.

- In summary duration is,
  - The slope of the price-yield curve.
  - A weighted average of the time till the cash flows will be received. (Macaulay Duration)
  - The approximate percentage change in price for a 1% change in yield. (Effective Duration)
Duration of a Portfolio

- Duration of a portfolio is the weighted average of the duration of the individual securities in the portfolio.
  \[
  \text{Portfolio Duration} = W_1D_1 + W_2D_2 + \ldots + W_ND_N
  \]

- The problem with the above equation is that it holds good only for a parallel shift in the yield curve. This is because securities with different maturities respond differently to a change in the yield.
Convexity is the measure of the curvature of a price-yield curve.

- Duration is an appropriate measure for small changes in the yield. For larger changes in yield, convexity should also be used.

  \[
  \text{Percentage Change in Price} = \text{Duration Effect} + \text{Convexity Effect} = [(-\text{Duration} \times \Delta y) + (\text{Convexity} \times \Delta y^2)] \times 100
  \]
Modified and Effective Convexity

• Just like duration, Effective convexity takes into account changes in cash flows due to options embedded in a bond which Modified Convexity ignores.

Price Value of a Basis Point (PVBP)

• Price Value of a basis point is the dollar change in the value of a bond for one basis change in the yield.

\[ \text{PVBP} = \text{Duration} \times 0.01\% \times \text{Bond Value}. \]