PRISTINE

Fixed Income Securities - II

Agenda

- Reading 64: Introduction to the Valuation of Debt Securities
- Reading 65: Yield Measures, Spot Rates, and Forward Rates
- Reading 66: Introduction to Measurement of Interest Rate Risk (1hr)
  - Key Issues in Measurement of Interest Rate Risk
  - Price Volatility and Convexity
  - Effective Duration
  - Bond's Modified Duration
  - Alternative Definition of Duration
  - Duration of Portfolio
  - Convexity Measure of Bond
  - Modified and Effective Convexity

Key Issues in Introduction to the Measurement of Interest Rate Risk

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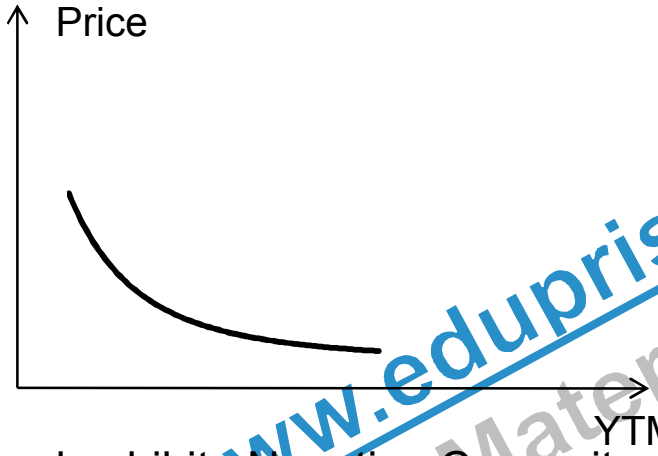
- Measuring Interest Rate Risk
- Price Volatility
- Convexity
- Effective Duration
- Alternative definitions of Duration
- Duration of a portfolio
- Convexity measure of a bond
- Modified and Effective Convexity
- Price Value of a Basis Point(PVBP)

Measuring Interest Rate Risk

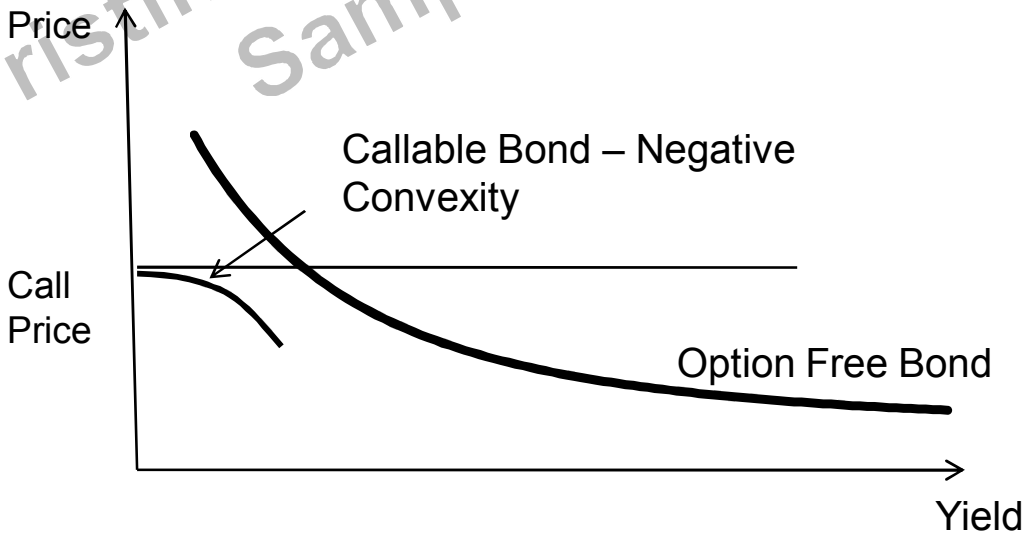
- Interest rate risk can be measured by two methods:
  - Full Valuation Method:
    - Under this method the normal valuation techniques are used to value a bond or a bond with embedded options.
    - When the interest rates change the entire exercise is repeated again to find out the value as per the new interest rates.
    - The two values are compared to arrive at the impact of change in interest rate.
  - Duration/Convexity Method: This gives an approximate result of the sensitivity of the bond. But it is much simples compared to the full valuation method.

Price Volatility and Convexity

- We have already seen that the price-yield curve is a negatively sloped and is a curve.

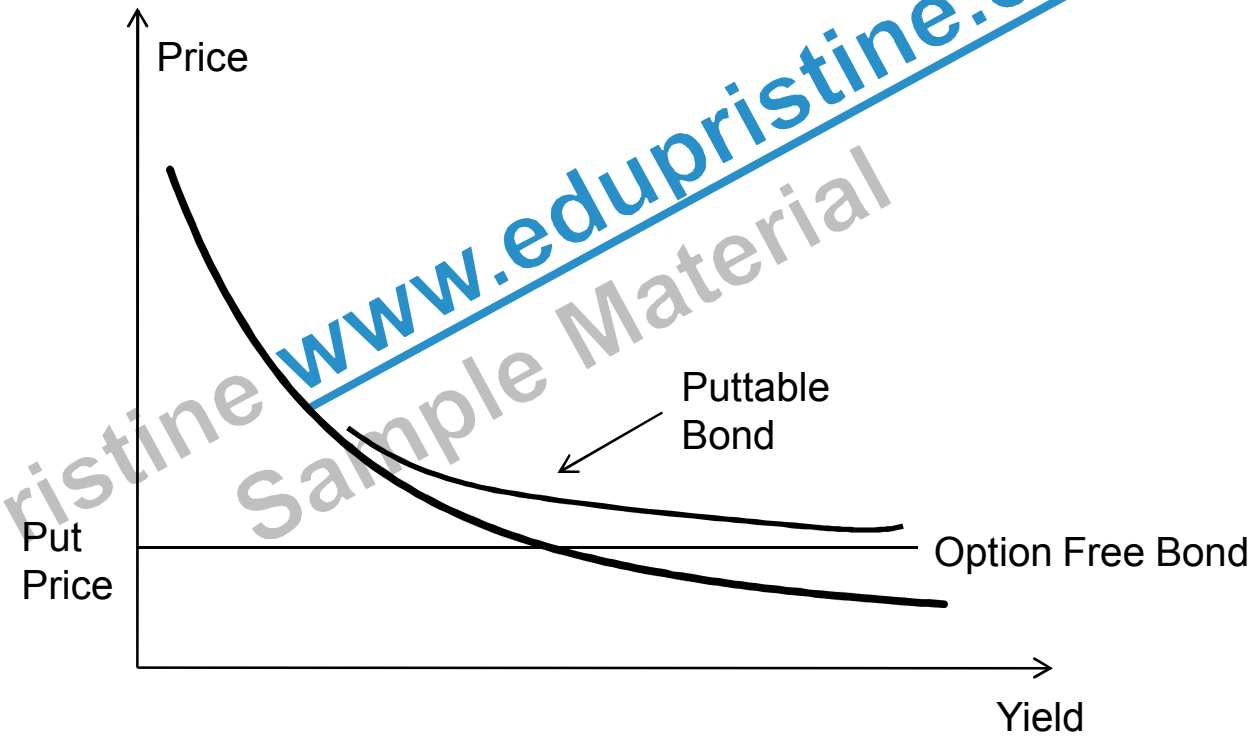


- The curve of a Callable bond exhibits Negative Convexity. This is because the increase in the price of a security as a result of fall in the yield is capped at the call price. See the below graph:



Price Volatility and Convexity

- The curve of a Puttable bond exhibits Positive Convexity. This is because the decrease in the price of a security as a result of increase in the yield is limited to the put price. See the below graph:



Effective Duration

- Duration is the measure of how long on an average the holder of the bond has to wait before he receives his payments on the bond. A coupon paying bond's duration would be lower than "n" as the holder gets some of his payments in the form of coupons before "n" years.
- Effective duration is calculated as:

$$\text{Effective Duration} = \frac{(\text{Bond price when yield falls} - \text{Bond price when yield rises})}{2 * (\text{Initial Price}) * (\text{Change in yield in decimals})}$$

Percentage change in Bond Price = -Effective Duration * Change in yield in percent. (Δy)

- Consider a bond trading at 96.54 with duration of 4.5 years. In this case

  ΔB = - 96.54* 4.5 Δy

  ΔB = -434.43 Δy

  If there is 10 basis points increase (+ Δy) in the yield then the bond price would change by:

  ΔB = -434.43 * (0.001)

  ΔB = -.43443

  Hence, B = 96.54- .43443 = 96.10

Bond modified duration calculation

Year	1	2	3	4	5	6	7	8	
Cash flows	5.125	5.125	5.125	5.125	5.125	5.125	5.125	105.1	
t*Ft	5.1	10.3	15.4	20.5	25.6	30.8	35.9	841.0	
(1+r)t+1	1.1	1.2	1.2	1.3	1.4	1.4	1.5	1.6	
	4.6	8.8	12.5	15.9	18.9	21.5	23.9	532.5	

Market rate	5.2%

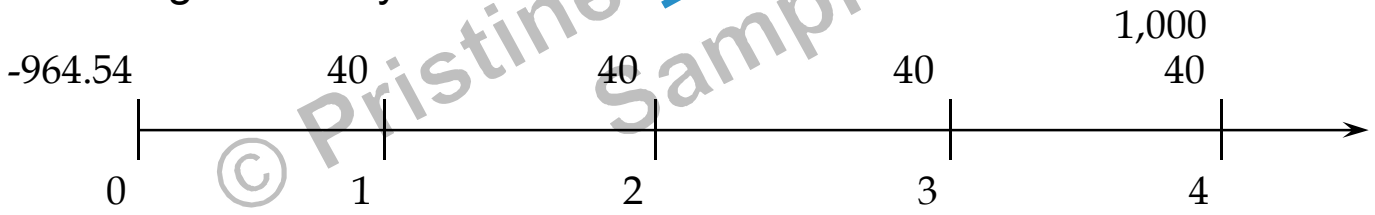
Price	**99.46**
Modified duration	**6.42**
Duration	**6.76**

Alternative definitions of Duration

- **Macaulay Duration:** is the weighted average of the times when the payments are made. And the weights are a ratio of the coupon paid at time "t" to the present bond price.
- Macaulay duration is also used to measure how sensitive a bond or a bond portfolio's price is to changes in interest rates.

$$\text{Macaulay Duration} = \frac{\sum_{t=1}^{n} \frac{t * C}{(1+y)^t} + \frac{n * M}{(1+y)^n}}{\text{Current Bond Price}}$$

- Where:
- t = Respective time period
- C= Periodic Coupon payments ; y =Periodic yield : n = Total number of periods
- M = maturity Value
- Calculating Macaulay Duration:



$$D = \frac{\frac{40}{(1.05)}(1) + \frac{40}{(1.05)^2}(2) + \frac{40}{(1.05)^3}(3) + \frac{1040}{(1.05)^4}(4)}{964.54} = \frac{3636.76}{964.54} = 3.77$$

Note that this is 3.77 six-month periods, which is about 1.89 years

Alternative definitions of Duration

- **Modified Duration:** is derived from Macaulay Duration. It is better than Macaulay Duration as it takes into account the current YTM.

$$\text{Macaulay Duration} = \frac{\text{Macaulay Duration}}{1 + \frac{r}{n}}$$

- **Effective Duration** calculations explicitly take into account the a bond's option provisions. The other methods of calculation ignore the option provision.

- In summary duration is,
  - The slope of the price-yield curve.
  - A weighted average of the time till the cash flows willl be received.(Macaulay Duration)
  - The approximate percentage change in price for a 1% change in yield.(Effective Duration)

Duration of a Portfolio

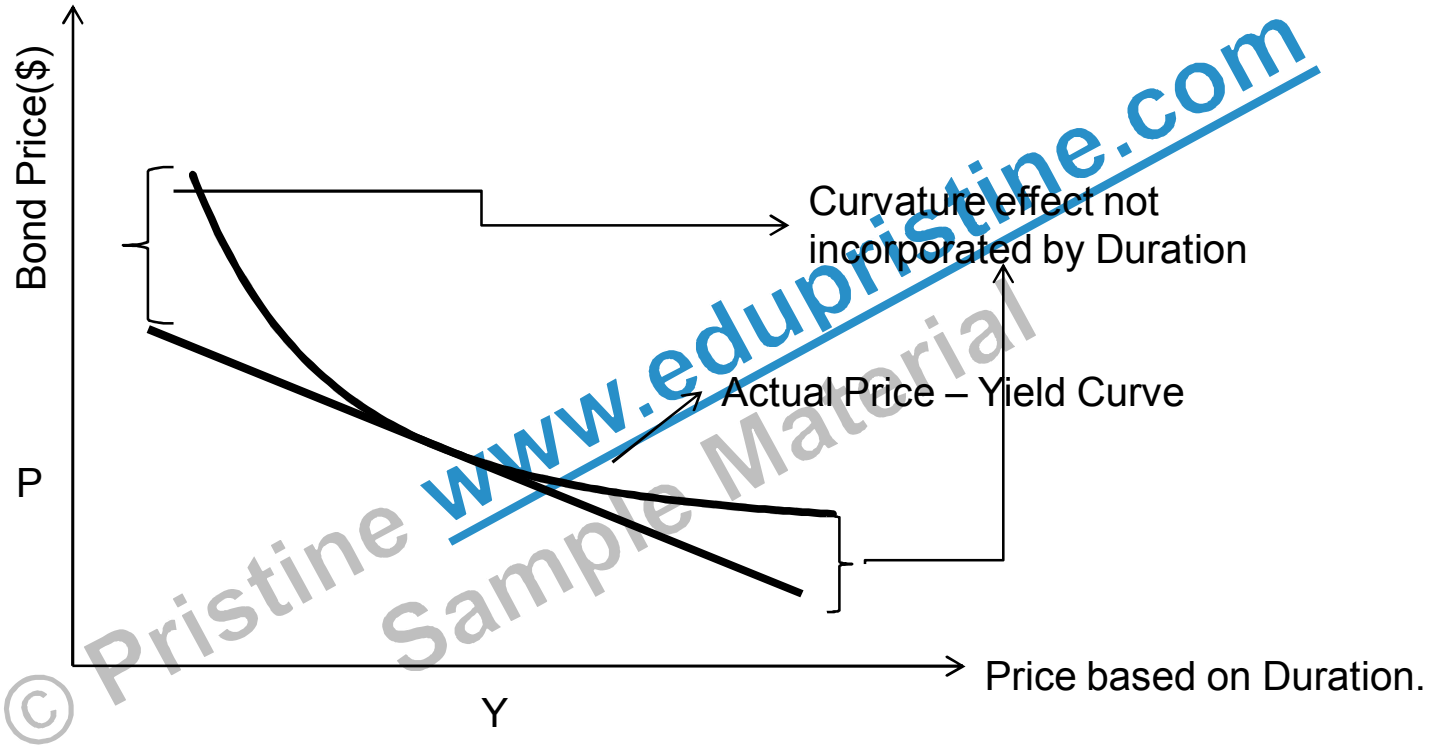
- Duration of a portfolio is the weighted average of the duration of the individual securities in the portfolio.

$$\text{Portfolio Duration} = W_1D_1 + W_2D_2 + ... + W_ND_N$$

- The problem with the above equation is that it holds good only for a parallel shift in the yield curve. This is because securities with different maturities respond differently to a change in the yield.

Convexity Measure of a Bond

Convexity is the measure of the curvature of a price-yield cuve.



- Duration is an appropriate measure for small changes in the yield. For larger changes in yield convexity should also be used.

  Percentage Change in Price = Duration Effect + Convexity Effect

  $=[(-\text{Duration} * \Delta y) + (\text{Convexity} * \Delta y^2)] * 100$

Modified and Effective Convexity

- Just like duration, Effective convexity takes into account changes in cash flows due to options embedded in a bond which Modified Convexity ignores.

Price Value of a Basis Point(PVBP)

- Price Value of a basis point is the dollar change in the value of a bond for one basis change in the yield.

  PVBP = Duration * 0.01% * Bond Value.