Key Issues in An Introduction to Portfolio Management

- Risk Aversion
- Assumptions of Markowitz model
- Expected return and Standard deviation
- Covariance
- Components of portfolio standard deviation
- Efficient Frontier
- Optimal Portfolio
Risk Aversion

• Risk Aversion simply means that investors prefer lesser risk.

• Assumptions of Markowitz Model
  – Investors are risk averse and try to minimise the risk and maximise return.
  – Investors look at each investment opportunity as a probability distribution of expected returns.
  – Investors are rational and behave in a manner as to maximise their utility with a given level of income or money.
  – Investors make investment decisions considering only the risk and return of the investment.
  – Investors measure risk as variance of returns.
**Expected return and Standard deviation**

**Individual Security:**
- Expected return of a single security is measured by multiplying the probability of achieving a return with the quantum of the return. The various returns are summed to arrive at the security's return.

\[ E(R) = P_1R_1 + P_2R_2 + P_3R_3 + \cdots + P_nR_n \]

- Variance (Standard deviation) of a single security

\[ \text{Variance} = \sigma^2 = \sum P_i [R_i - E(R)]^2 \]

**TWO ASSET PORTFOLIO:**

\[ E(R_p) = w_A E(R_A) + (1-w_A) E(R_B) \]

\[ \text{Variance} \ (w_A k_A + w_B k_B) = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2 w_A w_B \sigma_A \sigma_B \rho_{AB} \]
Covariance

- **Covariance**: measure the extent to which two change together.

\[ \rho_{X,Y} = \frac{\text{cov}(X, Y)}{\sigma_X \sigma_Y} = \frac{E((X - \mu_X)(Y - \mu_Y))}{\sigma_X \sigma_Y} \]

- **Correlation**: measures the strength of the linear relationship between two variables.
  - It is a better measure than Covariance as it is easier to interpret. It varies from +1 to -1.
  - \(<1\), variances of returns of a portfolio is less than a weighted average of the individual variances of the portfolio securities.
  - The lower the correlation between 2 securities the greater the diversification benefits.
Components of portfolio standard deviation

- Portfolio Variance is calculated using the below formula:

- For two-asset portfolio:
  \[ \text{Var}(w_A k_A + w_B k_B) = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2 w_A w_B \sigma_A \sigma_B \rho_{AB} \]
  - where,
    - \( \rho_{AB} \) is correlation coefficient between A and B.
    - \( w_A, w_B \) are weights of the asset A and B.

- If \( \rho = 1 \)
  \[ \text{Var}(w_A k_A + w_B k_B) = (w_A \sigma_A + w_B \sigma_B)^2 \]

- If \( \rho < 1 \)
  \[ \text{Var}(w_A k_A + w_B k_B) < (w_A \sigma_A + w_B \sigma_B)^2 \]

- So there is a risk reduction from holding a portfolio of assets if assets do not move in perfect unison.
Importance of Correlation

• $E(R_A) = 10\%, \sigma_A = 20\%, E(R_B) = 10\%, \sigma_B = 20\%$
• Assume the weights to be 50 % for A & B
• Calculate portfolio returns when
  – Case 1 : $\rho_{AB} = 1$,
  – Case 2 : $\rho_{AB} = 0$,
  – Case 3 : $\rho_{AB} = -1$

• Expected return = $10\% \times 0.5 + 10\% \times 0.5 = 10\%$ (in all three cases)
• Variance
  – Case 1 : $(0.5^2)(0.2^2) + (0.5^2)(0.2^2) + 2 \times 0.5 \times 0.5 \times 0.2 \times 0.2 \times 1 = 0.04$
  – Case 2 : $(0.5^2)(0.2^2) + (0.5^2)(0.2^2) + 2 \times 0.5 \times 0.5 \times 0.2 \times 0.2 \times 0 = 0.02$
  – Case 3 : $(0.5^2)(0.2^2) + (0.5^2)(0.2^2) + 2 \times 0.5 \times 0.5 \times 0.2 \times 0.2 \times -1 = 0.00$
Efficient Frontier refers to the set of portfolios which will give the highest return for each level of risk.
Optimal Portfolio

- Optimal Portfolio is most preferred portfolio of all the possible options.
- Optimal portfolio varies from investor to investor because a risk averse investor will prefer a portfolio with lower risk and thus a lower return whereas a risk taking investor will prefer a portfolio with higher risk and commensurately higher return.

- Combining this concept with the Efficient frontier:
  - Optimal portfolio for each investor is the point where his indifference curve is a tangent to the Efficient Frontier.