Derivatives – I
Forwards and Futures
Agenda

• Introduction, Asset types and derivatives
  • Valuation of Forward Contract
  • Forward Rate Agreement
  • Futures and Forwards on Currencies
Introduction

• Exchange traded Markets
  – Market where individuals trade standardized contracts that have been defined by the exchange themselves. Chicago Board of Trade and Chicago Mercantile Exchange are two examples
  – Open outcry system and Electronic trading

• Over the counter markets
  – Much larger than the exchange traded market in terms of value of underlying assets (at least 4 times larger)
  – Trades done between financial institutions or between financial institutions and clients. Financial institutions act as a market maker (quote both bid and offer)
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Pricing and Valuation of Forward Contracts

• Arbitrage free Forward prices are given as:
  \[ F(0,T) = S_0 e^{rT} \]

• If we long the forward contract at time \( t=0 \), at forward’s price \( F(0,T) \), the initial cash outlay would be zero.

• At time \( t=t \), we have claim on the asset which is worth \( S_t \) and an obligation to pay \( F(0,T) \) at time \( t=T \).

• At time \( t=T \), we pay \( F(0,T) \) and receive the asset worth \( S_T \).

<table>
<thead>
<tr>
<th>Long Forward Contract- ( F(0,T) )</th>
<th>Claim on asset worth ( S(t) )</th>
<th>Receive asset worth ( S(T) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outflow=0</td>
<td>Obligation to pay ( F(0,T) ) at ( T )</td>
<td>Outflow = ( F(0,T) )</td>
</tr>
<tr>
<td>( t=0 )</td>
<td>( t=t )</td>
<td>( t=T )</td>
</tr>
</tbody>
</table>
Forward Prices

• Forward rates are rates of interest implied by the current zero rates for a period of time in the future

• For example if we have the zero rates for year 4 and year 5 then the forward rate for the period of time between year 4 and year 5 would be known as the forward rate for that time period of 1 year.

\[
\begin{array}{c|c|c}
\text{Year 4} & \text{Year 5} \\
100 \times e^{0.04 \times 4} & 100 \times e^{0.05 \times 5} \\
F_4 = 4\% & F_5 = 5\% \\
F_{4,5} & \\
\end{array}
\]

• Consider that you invest $100 for 4 years and then roll it forward for one year in the 5 year. Then the total amount would be given as:

\[
100 \times e^{0.04 \times 4} e^{F_{4,5} \times 1}
\]

• If the same $100 was invested for 5 years instead then it would grow to

\[
100 \times e^{0.05 \times 5}
\]

• Equating the two we get

• \( F_{4,5} = 8.99\% \)
Forwards price

- The price of a forwards contract is given by the equation below:
  \[ F_0 = S_0 e^{rt} \]
  in the case of continuously compounded risk free interest rate, \( r \)
  \[ F_0 = S_0 (1+r)^t \]
  in the case of annual risk free interest rate, \( r \)
  where,
  \( F_0 \): forward price
  \( S_0 \): Spot price
  \( t \): time of the contract

- **Known income from underlying**
  If the underlying asset on which the forward contract is entered into provides an income with a present value \( I \), then the forward contract would be valued as:
  \[ F_0 = (S_0 - I)e^{rt} \]

- **Known yield from underlying**
  If the underlying asset on which the forward contract is entered into provides a continuously compounded yield \( q \), then the forward contract would be valued as:
  \[ F_0 = S_0 e^{(r-q)t} \]
  where \( q \) is continuously compounded \% of return on the asset.
Value of forward contracts

- At the time on entering into a forward contract, long or short, the value of the forward is zero
- This is because the delivery price (K) of the asset and the forward price today (F0) remains the same
- The value of the forward is basically the present value of the difference in the delivery price and the forward price
- Value of a long forward, f, is given by the PV of the pay off at time T:
  \[ f = (F_0 - K)e^{-rT} \]
  K is fixed in the contract, while F0 keeps changing on an everyday basis

- For continuous yielding underlying
  \[ f = S_0e^{-qt} - Ke^{-rt} \]
- For discrete dividend paying stock
  \[ f = S_0 - I - Ke^{-rt} \]
- **Index futures**: A stock index can be considered as an asset that pays dividends and the dividends paid are the dividends from the underlying stocks in the index
  If q is the dividend yield rate then the futures price is given as:
  \[ F_0 = S_0e^{(r-q)t} \]
- **Index Arbitrage**
  - When \( F_0 > S_0e^{(r-q)T} \) an arbitrageur buys the stocks underlying the index and sells futures
  - When \( F_0 < S_0e^{(r-q)T} \) an arbitrageur buys futures and shorts or sells the stocks underlying the index
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Forward rate agreements (FRAs)

- In general: 
  \[ F(T_1, T_2) = \frac{R_2T_2 - R_1T_1}{(T_2 - T_1)} \]

- A forward rate agreement (FRA) is an over the counter agreement where the forward interest rate, \( F_{t1,t2} \), is fixed for a certain principal between times \( T_1 \) and \( T_2 \).

- The payer of the fixed interest rate is also known as the borrower or the buyer. The buyer hedges against the risk of rising interest rates, while the seller hedges against the risk of falling interest rates.

- Payment to the long at settlement =
  \[ \text{Notional Principal} \times \frac{(\text{Rate at settlement} - \text{FRA Rate}) \times (\text{days}/360)}{1 + (\text{Rate at settlement}) \times (\text{days} / 360)} \]

- Example: FRA that settles in 30 days
  - $1 million notional
  - Based on 90-day LIBOR
  - Forward rate of 5%, Actual 90-day LIBOR at settlement is 6%

- \((6\% - 5\%) \times (90/360)\times $1m = $2,500\)
- Value at settlement: \( 2,500 / (1 + (90/360)\times6\%) = $2,463 \)
Forward vs Futures Prices

• Forward and futures prices are usually assumed to be the same. When interest rates are uncertain they are, in theory, slightly different:
• A strong positive correlation between interest rates and the asset price implies the futures price is slightly higher than the forward price
• A strong negative correlation implies the reverse
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Futures and Forwards on Currencies

- A foreign currency is analogous to a security providing a dividend yield
- The continuous dividend yield is the foreign risk-free interest rate
- It follows that if \( r_f \) is the foreign risk-free interest rate

\[
F_0 = S_0 e^{(r-r_f)T}
\]

- If the spot price is USD/INR then the forward price is also in USD/INR and the foreign risk-free rate is Indian risk free rate.

\[
\frac{F_{USD}}{INR} = \frac{S_{USD}}{INR} e^{(r_{USD}-r_{INR})T}
\]
Example

- The forward rate of a 3-month EUR/USD foreign exchange contract is 1.1565 USD per EUR. USD LIBOR is 4% and EUR LIBOR is 2%. The spot USD per EUR exchange rate is ?
Solution

- **Answer:**
  - $F_0 = S_0 e^{(r-r_f)t}$
  - $1.1565 e^{(.04 - .02) .25} = 1.1507$
Types of Futures

**Treasury Bill Futures**
Based on $1 million face value T-bills maturing in 90 days.
- Price quote = 100 – Annualized discount rate in %
- Cash settled

**Treasury Bond Futures**
- Traded for Treasury bonds with maturities >15 years.
- **Deliverable** contract.
- Face value of $100,000.
- Are quoted as a percent and fractions of 1% (measured in 1/32 nds) of face value.
- The Short position holder has an option to deliver any of the several bonds to satisfy the contract terms. This least expensive product is also known as the cheapest-to-deliver-bond.
Types of Futures (Cont...)

**Eurodollar Futures**
- $1 million face value
- Based on 90-day LIBOR
- Price quote = 100 – annualized LIBOR in %
- Cash settled
- Minimum change = 1 tick = 0.01% => $25
Types of Futures (Cont...)

**Stock Index Future**
- S&P 500 is the most popular Index Future
- Trades in Chicago
- Settles in cash as it is not possible to physically deliver the index
- Multiplier of 250
- Value of contract = 250 * index level

**Currency Future**
- Much smaller market than currency forwards
- Price is stated in USD/unit
  - E.g. Size of peso contract is MXP 225,000, euro contract is EUR 145,000
Example

• Assume that the current 1-year forward exchange rate is 1.200 USD per EUR. An American bank pays 2.4% annual interest rate on a 1-year deposit and a 4.0% annual interest rate on a 3-year USD deposit. A European bank pays a 1.5% annual interest rate for a 1-year deposit and a 2.0% annual interest rate for a 3-year EUR deposit. The forward exchange rate in USD per EUR for exchange three years from today is closest to:
Answer:

- The 2 year forward rate in US = \( \sqrt \left[ \frac{(1.04)^3}{1.024} \right] - 1 = 4.81\% \)
- The 2 year forward rate in Europe = \( \sqrt \left[ \frac{(1.02)^3}{1.015} \right] - 1 = 2.25\% \)
- The forward exchange rate in USD per EUR for exchange three years: \( 1.2 \times (1.0481^2) / (1.0225^2) = 1.261 \)