Key Issues in An Introduction to Portfolio Management

- Risk Aversion
- Assumptions of Markowitz model
- Expected return and Standard deviation
- Covariance
- Components of portfolio standard deviation
- Efficient Frontier
- Optimal Portfolio
Risk Aversion

• **Risk Aversion**: means that investors prefer lesser risk (from two stocks with same risk an investor will prefer a stock with higher return & from two stocks with same return an investor will prefer a stock will less risk)

• **Assumptions of Markowitz Model**
  – Investors are risk averse and try to minimize the risk and maximize return.
  – Investors look at each investment opportunity as a probability distribution of expected returns.
  – Investors are rational and behave in a manner as to maximize their utility with a given level of income or money.
  – Investors make investment decisions considering only the risk and return of the investment.
  – Investors measure risk as variance of returns.
Expected return and Standard deviation

Symbols used:
- \( P \) represents probability, \( R \) represents return
- standard deviation is represented with \( \sigma \) & variance is represented with \( \sigma^2 \)
- covariance of stock A & B is represented with \( \rho_{AB} \)

- **Expected return** of a single security is measured by *multiplying the probability* of achieving a return with the *quantum* of the *corresponding return*. The various return are summed to arrive at the security

\[
E(R) = P_1R_1 + P_2R_2 + P_3R_3 + \ldots + P_nR_n
\]

- Variance(square of the standard deviation) of a single security:

\[
\text{Variance} = \sigma^2 = \sum P_i[R_i - E(R)]^2
\]

- **Two Asset Portfolio**: (Here A & B represents stock A & B)

\[
E(R_p) = w_AE(R_A) + (1-w_A)E(R_B)
\]

\[
\text{Variance (of portfolio of stock A & B)} = w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + 2w_Aw_B\sigma_A\sigma_B\rho_{AB}
\]

Note: Above two formulae are very important
Covariance

- **Covariance**: measure the extent to which two stocks change together

\[
\rho_{X,Y} = \frac{\text{cov}(X, Y)}{\sigma_X \sigma_Y} = \frac{E((X - \mu_X)(Y - \mu_Y))}{\sigma_X \sigma_Y}
\]

where mean is represented by \( \mu \)

- **Correlation**: measures the strength of the **linear relationship** between two variables
  - It is a better measure than Covariance as it is easier to interpret. It varies from +1 to -1
  - Positive correlation shows that the stocks are moving in the same direction
  - Negative correlation shows that the stocks are moving in opposite direction
  - \(<1\), variances of returns of a portfolio is less than a weighted average of the individual variances of the portfolio securities
  - The lower the correlation between 2 securities the greater the diversification benefits
Components of portfolio standard deviation

- Portfolio Variance is calculated using the below formula:

  - For two-asset portfolio:
    \[ \text{Var}(w_A k_A + w_B k_B) = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2 w_A w_B \sigma_A \sigma_B \rho_{AB} \]

  - where,
    - \( \rho_{AB} \) is correlation coefficient between A and B.
    - \( w_A, w_B \) are weights of the asset A and B.

  - If \( \rho = 1 \)
    \[ \text{Var}(w_A k_A + w_B k_B) = (w_A \sigma_A + w_B \sigma_B)^2 \]

  - If \( \rho < 1 \)
    \[ \text{Var}(w_A k_A + w_B k_B) < (w_A \sigma_A + w_B \sigma_B)^2 \]

  - So there is a risk reduction from holding a portfolio of assets if assets do not move in perfect unison.
Importance of Correlation

- \( E(R_A) = 10\% \), \( \sigma_A = 20\% \), \( E(R_B) = 10\% \), \( \sigma_B = 20\% \)
- Assume the weights to be 50% for A & B
- Calculate portfolio returns when
  - Case 1 : \( \rho_{AB} = 1 \),
  - Case 2 : \( \rho_{AB} = 0 \),
  - Case 3 : \( \rho_{AB} = -1 \)

  \[ \text{Expected return} = 10\% \times 0.5 + 10\% \times 0.5 = 10\% \text{ (in all three cases)} \]

- Variance
  - Case 1 : \((0.5^2) \times (0.2^2) + (0.5^2) \times (0.2^2) + 2 \times 0.5 \times 0.5 \times 0.2 \times 0.2 \times 1 = 0.04 \)
  - Case 2 : \((0.5^2) \times (0.2^2) + (0.5^2) \times (0.2^2) + 2 \times 0.5 \times 0.5 \times 0.2 \times 0.2 \times 0 = 0.02 \)
  - Case 3 : \((0.5^2) \times (0.2^2) + (0.5^2) \times (0.2^2) + 2 \times 0.5 \times 0.5 \times 0.2 \times 0.2 \times (-1) = 0.00 \)
Efficient Frontier

- Efficient Frontier refers to the set of portfolios which will give the highest return for each level of risk.
Optimal Portfolio

- **Optimal Portfolio** is most preferred portfolio of all the possible options
- Optimal portfolio **varies from investor to investor** because a risk averse investor will prefer a portfolio with lower risk and thus a lower return whereas a risk taking investor will prefer a portfolio with higher risk and commensurately higher return.

- Combining this concept with the **Efficient frontier**:
  - Optimal portfolio for each investor is the **point** where his **indifference curve is a tangent to the Efficient Frontier**