Normal Distribution

- Normal distribution is a **continuous, symmetric probability distribution** that is completely described by two parameters: its mean, $\mu$, and its variance, $\sigma^2$.

- General Normal random variable – $X \sim N(\mu, \sigma^2)$
  - The normal distribution is said to be bell-shaped with the mean showing its central location and the variance showing its “spread”.
  - A linear combination of two or more Normal random variables is also normally distributed.

- Standard Normal distribution – $Z \sim N(0, 1)$.
  - Is a Normal distribution with mean $\mu=0$, and variance $\sigma^2=1$.

- **Univariate** distribution: It is the distribution of a **single random variable**

- **Multivariate** distribution: It specifies the probabilities associated with a **group of random variables** & is meaningful only when the behavior of each random variable in the group is in some way dependent upon the behavior of the others
Effect of Varying Parameters ($\mu_x$ & $\sigma_x$)

- $\mu^A = \mu^B$, $\sigma^A > \sigma^B$
- $\mu^A < \mu^C$, $\sigma^A = \sigma^C$
Normal Distribution Random Variable

- General Normal random variable \( X \sim N(\mu, \sigma^2) \)
  - \( X \) can be standardized to a Standard Normal random variable.
  - Resulting variable has mean zero and variance equal to 1.

\[
Z = \frac{X - \mu}{\sigma}
\]

- Calculating probabilities for a normal random variable:
  - \( X \sim N(\mu, \sigma^2) \) taking on a range of specified values, say \( a < X < b \), directly as the area under the normal curve using the cumulative Normal distribution function as:

\[
N(a < X < b| \mu, \sigma^2) = N(X < b| \mu, \sigma^2) - N(X < a| \mu, \sigma^2).
\]
  - You should be able to show what this looks like using a diagram of the Normal distribution.
The standard normal distribution has mean = 0 and standard deviation sigma=1
Confidence Intervals

\[ X = \mu_x \pm Z \sigma_x \]

90% level: \[ \mu_x - 1.96\sigma_x \text{ to } \mu_x + 1.96\sigma_x \]

95% level: \[ \mu_x - 1.65\sigma_x \text{ to } \mu_x + 1.65\sigma_x \]

99% level: \[ \mu_x - 2.58\sigma_x \text{ to } \mu_x + 2.58\sigma_x \]