Introduction to Risk

- Risk can be broadly defined as the degree of uncertainty about future net returns
  - Credit risk relates to the potential loss due to the inability of a counterpart to meet its obligation
  - Operational risk takes into account the errors that can be made in instructing payments or settling transactions
  - Liquidity risk is caused by an unexpected large and stressful negative cash flow over a short period
  - Market risk estimates the uncertainty of future earnings, due to the changes in market conditions

- Broadly the standard deviation of the variable measures the degree of risk inherent in the variable.
- Say the standard deviation of returns from the assets owned by you is 50% and the standard deviation of returns from assets I own is 0%. We can say that risk of my assets is zero.

I own risk-less assets as the standard deviation of returns of my assets is 0%

My assets are very risky as the standard deviation of returns of my assets is 50%.
Value at Risk (VAR)

• Value at Risk (VaR) has become the standard measure that financial analysts use to quantify this risk.
• VAR represents **maximum potential loss** in value of a portfolio of financial instruments with a **given probability** over a **certain time horizon**.
• In simpler words, it is a number that indicates how much a financial institution can lose with probability (p) over a given time horizon (T).

• Say the 95% daily VAR of your assets is $120, then it means that out of those 100 days there would be 95 days when your daily loss would be less than $120. This implies that during 5 days you may lose more than $120 daily.

> There may be a day out of 100 when your loss is $5000, which means VAR doesn’t tell anything about the extent to which we can lose
Visualizing VAR

The colored area of the normal curve constitutes 5% of the total area under the curve.

There is 5% probability that the losses will lie in the colored area i.e. more than the VAR number.

<table>
<thead>
<tr>
<th>Confidence (x%)</th>
<th>Z_x%</th>
</tr>
</thead>
<tbody>
<tr>
<td>90%</td>
<td>1.28</td>
</tr>
<tr>
<td>95%</td>
<td>1.65</td>
</tr>
<tr>
<td>97.5%</td>
<td>1.96</td>
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<tr>
<td>99%</td>
<td>2.32</td>
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Measuring Value-at-Risk (VAR)

- $Z_{X\%}$: the normal distribution value for the given probability (x%) (normal distribution has mean as 0 and standard deviation as 1)
- $\sigma$: standard deviation (volatility) of the asset (or portfolio)

- VAR in absolute terms is given as the product of VAR in % and Asset Value:

$$VAR = VAR_{X\%} \times \sigma \times \text{Asset Value}$$

- This can also be written as:

$$VAR = Z_{X\%} \times \sigma \times \text{Asset Value}$$
Measuring Value-at-Risk (VAR)

- VAR for n days can be calculated from daily VAR as:

\[
\text{VaR}^{(\text{ndays})}(\text{in}\%) = \text{VaR}^{(\text{daily VaR})}(\text{in}\%) \times \sqrt{n}
\]

- This comes from the known fact that the n-period volatility equals 1-period volatility multiplied by the square root of number of periods (n).

\[
\text{VaR}^{(\text{ndays})}(\text{in}\%) = Z_{\times \%} * \sigma * \text{Asset Value} * \sqrt{n}
\]

- As the volatility of the portfolio can be calculated from the following expression:

\[
\sigma_{\text{portfolio}} = \sqrt{w_a^2 \sigma_a^2 + w_b^2 \sigma_b^2 + 2w_a w_b \sigma_a \sigma_b \rho_{ab}}
\]

- The above written expression can also be extended to the calculation of VAR:

\[
\text{VaR}^{(\text{portfolio})}(\text{in}\%) = \sqrt{w_a^2 (\% \text{VAR}_a)^2 + w_b^2 (\% \text{VAR}_b)^2 + 2w_a w_b (\% \text{VAR}_a)(\% \text{VAR}_b) \rho_{ab}}
\]
Question 1

- Asset daily standard deviation is 1.6%
- Market Value is USD 10 mn
- What is VaR (%) at 99% confidence?

Solution: Daily VaR = 0.016 x 10 x 2.33 = 0.3728 mn
Question 2

• What is the VaR value for 10 day VaR in the earlier case?

• Solution: 10 day VaR = 0.3728 x (10)^0.5 = 1.1789
Question 3

- What is the daily portfolio VaR at 97.5% confidence level?
  - Investment in asset A is Rs. 40 mn
  - Investment in asset B is Rs. 60 mn
  - Volatility of asset A is 5.5% and asset B is 4.25%
  - Portfolio VaR if correlation between A and B is 20% ?

Solution:
VaR(A)(in %) = 5.5 x 1.96 = 10.78%; VaR(B)(in %) = 4.25 x 1.96 = 8.33%;

Portfolio VaR = [(40 x 0.1078)^2 + (60 x 0.0833)^2 + 2 x 0.1078 x 0.833 x 40 x 60 x 0.20]^{0.5}
= 7.22 mn
Extended Question 3.1

• Portfolio VaR if
  – If correlation between A and B is Zero?
  – What if correlation is 1?
  – Or if -1?
• What are the implications?
Question 4

- Market Value of asset Rs. 10 mn
- Daily variance is 0.0005
- What is the annual VaR at 95% confidence with 250 trading days in a year?

Solution;

Daily VaR = 10 x (0.0005)\(^{0.5}\) x 1.65 = 0.36895 mn
Annual VaR = 0.36895 x (250)^{0.5} = 5.834 mn
Question 5

- For an uncorrelated portfolio what is the VaR if:
  - VaR asset A is Rs 10 mn
  - VaR asset B is Rs. 20 mn

Solution: This would require weights of the assets. Assuming it to be 50-50, the VaR comes out to be 11.18 mn