Counting Principle

- Number of ways of selecting r objects out of n objects
  - ^nC_r
  - n!/(r!)*(n-r)!

- Number of ways of giving r objects to n people, such that repetition is allowed
  - N^r

Question-Counting Principle

- In how many ways 3 stocks can be chosen out of 10 stocks in a portfolio?(Combination)
  - Choosing 3 out of 10 stocks is basically the number of combinations of 3 objects out of 10
  - Therefore, the number of ways are $^{10}C_3 = \frac{10!}{3!*(10-3)!} = 120$

- In how many ways 3 stocks can be sold, if the sold stock is bought back in the portfolio before the next stock is sold?
  - First stock can be sold in 10 ways
  - Second can be sold in again 10 ways
  - Third stock can again be sold in 10 ways
  - Therefore total number of ways become $=10^3 = 1000$

Question

- You wish to choose a portfolio of 3 bonds and 4 stocks from a list of 5 bonds and 8 stocks. How many different 7 asset portfolio can you make from this list.
  - 80
  - 700
  - 1,716
  - 100,800

Solution

- B
  - Solution:

$$^{5}C_{3} * ^{8}C_{4} = \frac{5!}{3!(5-3)!} * \frac{8!}{4!(8-4)!} = \frac{5*4}{2*1} * \frac{8*7*6*5}{4*3*2*1} = 10*70 = 700$$

Question

- There are 10 sprinters in the Olympic finals. How many ways can the gold, silver, and bronze medals be awarded?
  - 120
  - 720
  - 1,440
  - 604,800

Solution

- B
  - $^{10}P_3 = 720$
  - Please note that this is a case of Permutation and not Combination.

Probability - Definitions

- A probability experiment involves performing a number of trials to enable us to measure the chance of an event occurring in the future. A trial is a process by which an outcome is noted.

- Examples of Definitions:
  - **Experiment:** Roll a die two hundred times noting the outcomes
  - **Event of interest:** A six faces upwards
  - **Trial:** Roll the die once
  - **Number of trials**: 200
  - **Outcomes:** 1, 2, 3, 4, 5 or 6

- Probability of an event to occur is defined as number of cases favorable for the event, over the number of total outcomes possible in unbiased experiment
  - For example, if the event is "occurrence of an even number when a die is rolled“
  - The probability is given by 3/6=1/2, since 3 faces out of the 6 have even numbers and each face has the same probability of appearing.

Conditional & Joint Probability

- **Joint Probability**
  - A statistical measure where the likelihood of two events occurring together and at the same point in time are calculated. Joint probability is the probability of event Y occurring at the same time event X occurs.
  - Notation for joint probability takes the form:

    P(X ∩Y) or P(X,Y)

    Which reads the joint probability of X and Y.
  - The following table shows the joint probability of different events. Let's say an economist is predicting the market scenario and the price of IBM stock from the next year.
  - Next year market can be Good, Bad or Neutral
  - IBM stock may go up or go down

Joint Probability Table:

Market / IBM	Good	Bad	Neutral	Total
UP	10%	30%	5%	*45%*
DOWN	0%	15%	40%	*55%*
Total	*10%*	*45%*	*45%*	***100%***

  - The probability of IBM stock being Up and Market being Good is 10%
  - Similarly, the probability of IBM stock being down and Market being neutral is 40%

Conditional & Joint Probability

- **Conditional Probability**
  - Probability of an event or outcome based on the occurrence of a previous event or outcome. Conditional probability is calculated by multiplying the probability of the preceding event by the updated probability of the succeeding event
  - The probability of event A given that the event B has occurred is P(A/B), which is equal to the ratio of joint probability of A and B, and unconditional probability of B.

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

- The unconditional probability of market being Neutral is 45%. Then using the table below we can find 3 conditional probabilities.
  - P(Up/Neutral) = 0.05 / 0.45
  - P(Up/Good) = 0.1/0.1
  - P(Down/Bad) = 0.15/0.45

Joint Probability Table:

Market / IBM	Good	Bad	Neutral	Total
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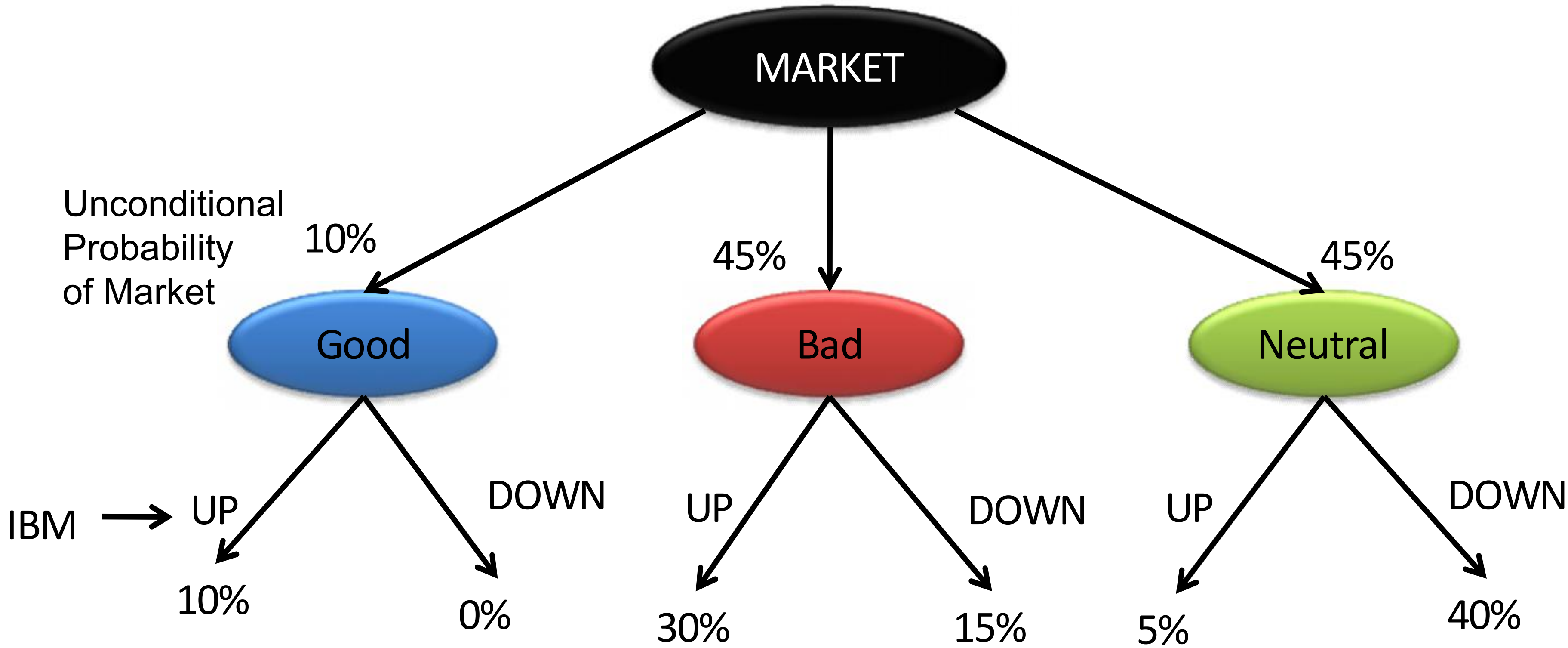
Total	*10%*	*45%*	*45%*	***100%***

Unconditional Probabilities of IBM stock being UP/Down

Conditional and Unconditional Probabilities



Calculate:
Unconditional Probability of market to be good next year?
Conditional Probability of IBM stock rising when the market is neutral?
Conditional Probability of market being good when IBM stock is down?

Some definitions and properties of Probability

- Definitions
  - **Mutually Exclusive:** If one event occurs, then other cannot occur
  - **Exhaustive:** all exhaustive events taken together form the complete sample space (Sum of probability = 1)
  - **Independent Events:** One event occurring has no effect on the other event

- The probability of any event A: $P(A) \in [0,1]$

- If the probability of happening of event A is P(A), then the probability of A not happening is (1-P(A)).
  - For example, if the probability of a company going bankrupt within one year period is 20%, then the probability of company surviving within next one year period is 80%.

$$P(A') = 1 - P(A)$$

Question

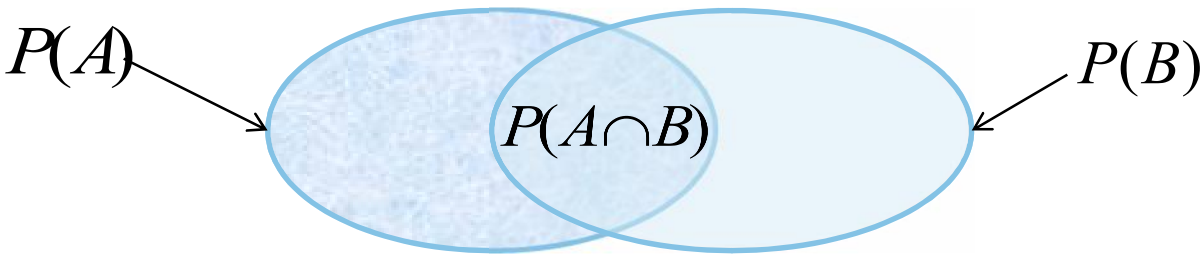
- For a bond with "B" rating, assume 1 year probability of default for each issuer is 6%, and that default probability of each issuer are independent. What is the probability that both issuers avoid default during the 1st year.
  - 88%
  - 88.4%
  - 94%
  - 96.4%

Solution

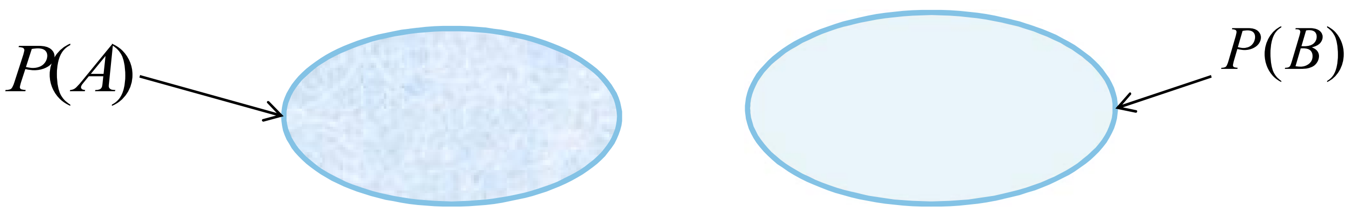
- B
  - Both would avoid default only if None defaults
  - This implies that first does not default AND second does not default
  - = (1 – PD (first)) x (1 – PD (second))
  - = (1 – 0.06) x (1 – 0.06) = 0.884 = 88.4%

Some Properties of Probability

- The probability of happening of event A or event B can be given as the sum of the three portions defined by the figure below.



$$P(A \cup B) = \begin{cases} P(A) + P(B) - P(A \cap B) \\ P(A) + P(B) \text{ if A and B are Mutually Exclusive} \end{cases}$$



$$P(A \cap B) = 0$$

Question

- Jensen, a portfolio manager is managing two portfolios. One for High Net Worth Individuals (HNI) and second for Low Net Worth Individuals (LNI).
- HNI portfolio contains 5 bonds and 7 stocks and LNI contains 6 bonds and 11 stocks.
- One instrument from HNI is transferred to LNI portfolio.
- Now Jensen selects an instrument from LNI, what is the probability that instrument selected is bond?

- 0.5382
- 0.7821
- 0.6435
- None of these

Solution

- B
  - Here required probability = [P(stock transferred from HNI) AND P(Stock selected from LNI)] OR [P(bond transferred from HNI) AND P(Stock selected from LNI)]
  - So, the required probability = $(7/12) \times (12/18) + (5/12) \times (11/18) = 139/216 = 0.6435$
  - Hence option ‘B’ is correct.

Sum Rule & Bayes' Theorem

- The unconditional probability of event B is equal to the sum of joint probabilities of event (A,B) and the probability of event (A',B). Here A' is the probability of not happening of A.
  - The joint probability of events A and B is the product of conditional probability of B, given A has occurred and the unconditional probability of event A.

$$P(B) = P(A \cap B) + P(A^c \cap B) = P(B/A)P(A) + P(B/A^c)P(A^c)$$

  - We know that P(AB) = P(B/A) * P(A)
  - Also P(BA)= P(A/B) * P(B)
  - Now equating both P(AB) and P(BA) we get:

$$P(A/B) = \frac{P(B/A) * P(A)}{P(B)}$$

  - P(B) can be further broken down using sum rule defined above:

$$P(A/B) = \frac{P(B/A)P(A)}{P(B/A)P(A) + P(B/A^c)P(A^c)}$$

Question

- Out of a group of 100 patients being treated for chronic back trouble, 25% are chosen at random to receive a new, experimental treatment as opposed to the more usual muscle relaxant-based therapy which the remaining patients receive. Preliminary studies suggest that the probability of a cure with the standard treatment is 0.3, while the probability of a cure from the new treatment is 0.6.

- How many patients (on an average) out of the 100 patients selected at random would be cured?
  - 30
  - 40
  - 37.5
  - 42.5

- Some time later, one of the patients returns to thank the staff for her complete recovery. What is the probability that she was given the new treatment?
  - 0.375
  - 0.425
  - 0.4
  - 0.425

Solution

- C
  - 25% are given new treatment =>75% are given old treatment.
  - P(Cure) = P(Cure/New) * P(New) + P(Cure/Old) * P(Old) = 0.375
  - So out of 100 patients 37.5 will get cured

- C
  - Apply Bayes' Theorem
  - P (New / Cure) = P (Cure / New) * P (New) / P (Cure) = 0.6 * 0.25 / 0.375 = 0.40

Question

- Calculate the probability of a subsidiary and parent company both defaulting over the next year. Assume that the subsidiary will default if the parent defaults, but the parent will not necessarily default if the subsidiary defaults. Assume that the parent had a 1 year PD = .5% and the subsidiary has 1 year PD of .9%.

  - 0.45%
  - 0.5%
  - 0.545%
  - 0.55%

Solution

- **B**
  - $P(S|P) = 1 = P(P \& S)/P(P)$
  - $P(P \& S) = P(P) = 0.5\%$