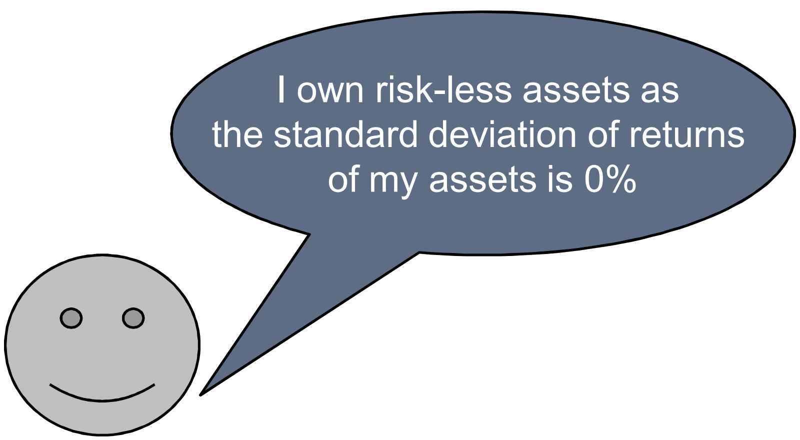
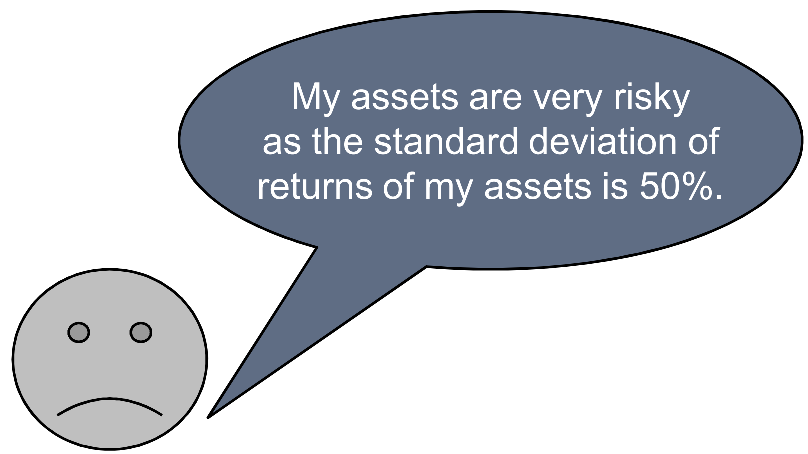
Introduction to Risk

- Risk can be broadly defined as the degree of uncertainty about future net returns
  - Credit risk relates to the potential loss due to the inability of a counterpart to meet its obligation
  - Operational risk takes into account the errors that can be made in instructing payments or settling transactions
  - Liquidity risk is caused by an unexpected large and stressful negative cash flow over a short period
  - Market risk estimates the uncertainty of future earnings, due to the changes in market conditions

- Broadly the standard deviation of the variable measures the degree of risk inherent in the variable.
- Say the standard deviation of returns from the assets owned by you is 50% and the standard deviation of returns from assets I own is 0%. We can say that risk of my assets is zero.





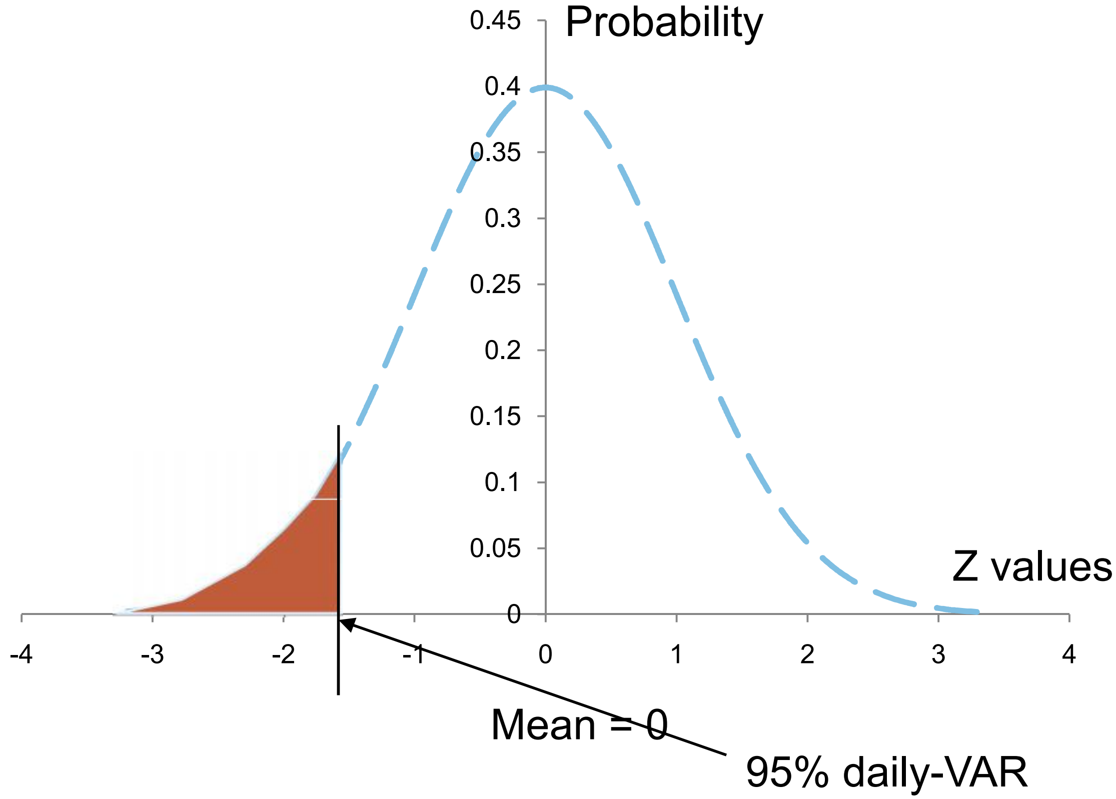
Value at Risk (VAR)

- Value at Risk (VaR) has become the standard measure that financial analysts use to quantify this risk.
- VAR represents **maximum potential loss** in value of a portfolio of financial instruments with a **given probability** over a **certain time horizon**.
- In simpler words, it is a number that indicates how much a financial institution can lose with probability (p) over a given time horizon (T).

- Say the 95% daily VAR of your assets is $120, then it means that out of those 100 days there would be 95 days when your daily loss would be less than $120. This implies that during 5 days you may lose more than $120 daily.

There may be a day out of 100 when your loss is $5000,
which means VAR doesn’t tell anything about the extent to which we can lose

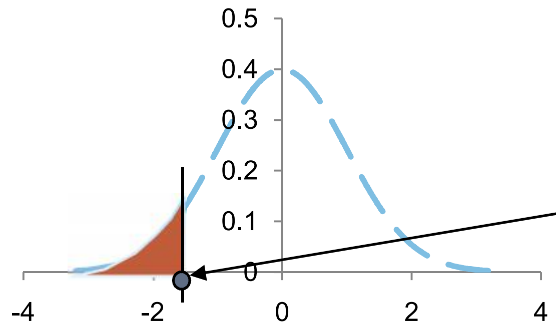
Visualizing VAR



Confidence (x%)	$Z_{X\%}$
90%	1.28
95%	1.65
97.5%	1.96
99%	2.32

- The colored area of the normal curve constitutes 5% of the total area under the curve.
- There is 5% probability that the losses will lie in the colored area i.e. more than the VAR number.

Measuring Value-at-Risk (VAR)



$$VAR_{X\%}(in\ \%) = Z_{X\%} * \sigma$$

- $Z_{X\%}$: the normal distribution value for the given probability (x%) (normal distribution has mean as 0 and standard deviation as 1)
- σ : standard deviation (volatility) of the asset (or portfolio)

- VAR in absolute terms is given as the product of VAR in % and Asset Value:

$$VAR = VAR_{X\%}(in\ \%) * AssetValue$$

- This can also be written as:

$$VAR = Z_{X\%} * \sigma * AssetValue$$

Measuring Value-at-Risk (VAR)

- VAR for n days can be calculated from daily VAR as:

$$VaR_{(n\,days)}(in\%) = VaR_{(daily\,VaR)}(in\%) * \sqrt{n}$$

- This comes from the known fact that the n-period volatility equals 1-period volatility multiplied by the square root of number of periods(n).

$$VaR_{(n\,days)}(in\%) = Z_{X\%} * \sigma * Asset\,Value * \sqrt{n}$$

- As the volatility of the portfolio can be calculated from the following expression:

$$\sigma_{portfolio} = \sqrt{w_a^2\,\sigma_a^2 + w_b^2\,\sigma_b^2 + 2w_a w_b * \sigma_a * \sigma_b * \rho_{ab}}$$

- The above written expression can also be extended to the calculation of VAR:

$$VaR_{portfolio}(in\%) = \sqrt{w_a^2\,(\%VAR_a)^2 + w_b^2\,(\%VAR_b)^2 + 2w_a w_b * (\%VAR_a) * (\%VAR_b) * \rho_{ab}}$$

Question 1

- Asset daily standard deviation is 1.6%
- Market Value is USD 10 mn
- What is VaR (%) at 99% confidence?

- Solution: Daily VaR = 0.016 x 10 x 2.33 = 0.3728 mn

Question 2

- What is the VaR value for 10 day VaR in the earlier case?
- Solution: 10 day VaR = 0.3728 x (10)^0.5 = 1.1789

Question 3

- What is the daily portfolio VaR at 97.5% confidence level?
  - Investment in asset A is Rs. 40 mn
  - Investment in asset B is Rs. 60 mn

  - Volatility of asset A is 5.5% and asset B is 4.25%
  - Portfolio VaR if correlation between A and B is 20% ?

Solution:

VaR(A)(in %) = 5.5 x 1.96 = 10.78%; VaR(B)(in %) = 4.25 x 1.96 = 8.33%;

Portfolio VaR = $[(40 \times 0.1078)^2 + (60 \times 0.0833)^2 + 2 \times 0.1078 \times 0.833 \times 40 \times 60 \times 0.20]^{0.5}$

= 7.22 mn

Extended Question 3.1

- Portfolio VaR if
  - If correlation between A and B is Zero?
  - What if correlation is 1 ?
  - Or if -1 ?
- What are the implications ?

Question 4

- Market Value of asset Rs. 10 mn
- Daily variance is 0.0005
- What is the annual VaR at 95% confidence with 250 trading days in a year?

- Solution;
  Daily VaR = 10 x $(0.0005)^{0.5}$ x 1.65 = 0.36895 mn
  Annual VaR = 0.36895 x $(250)^{0.5}$ = 5.834 mn

Question 5

- For an uncorrelated portfolio what is the VaR if:
  - VaR asset A is Rs 10 mn
  - VaR asset B is Rs. 20 mn

Solution: This would require weights of the assets. Assuming it to be 50-50, the VaR comes out to be 11.18 mn