

 **Quantitative Analysis**

nCr

n!/(r!)\*(n-r)!





Number of ways of selecting the ‘r’ objects out of ‘n’ objects:

Nr



Number of ways of giving r objects to n people, such that repetition is allowed:

experiment involves performing a number of trials to enable us to measure the chance of an event occurring in the future. A trial is a process by which an outcome is noted.

**probability**

 **A**

**Probability**

* **Probability** of an event to occur is defined as number of cases favorable for the event, over the number of total outcomes possible in unbiased experiment.
* Event A – Rolling an even number
* Event A possible outcomes: {2,4,6}
* P(A) = # ways of to roll an even number / Total # of sides =

{2,4,6}/{1,2,3,4,5,6} = 50%

* Event B – Rolling a 6
* P(B) = # ways to roll a 6/ Total # of sides = 1/6
* Event C – Rolling a number less than 3, i.e. {1,2}
* P(C) = 2/6

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| **Let us consider a Problem:** A single 6-sided die is rolled 100 times, and the outcomes are noted. What is the probability of each outcome? What is the probability of rolling an even number? of rolling an odd number? |

#### An Experiment is a situation involving chance or probability that leads to results called outcomes.

**Example:** In the above problem, rolling the dice is the experiment

An **outcome** is the result of a single trial of an experiment.

**Example:** when we roll the dice one, the possible outcomes are 1,2,3,4,5, or 6

An **event** is a set of outcomes of an experiment (a subset of total possible outcomes).

**Example:** One event ‘A’ can be defined as getting an even number when we roll the dice

A **trial** constitutes rolling the die once. The experiment can have several trials, like 100 or 200

**Independent Events:** One event occurring has no effect on the other event, i.e., we have more than 1 random variable.

Event A: Kohli scores a century in the next match, P(A) = 10% Event B: Stock market is up tomorrow, P(B) = 8%

P(A and B) = P(A ∩ B) = P(A) x P(B)  Also called, ‘Joint Probability’

Since the two events are independent, the probability that Kohli gets a century, and the Stock market goes up is 10% x 8% = 0.8%

What if A and B are dependent?

Event A: Kohli scores a century in the next match, P(A) = 10% Event B: India wins the match, P(B) = 40%

P(A and B) = ?

Here, conditional probability comes into picture.

### Example

**Mutually Exclusive events:** If one event occurs, then another event cannot occur. For a given random variable, the probability of any of two mutually exclusive events occurring is just the sum of their individual probabilities. **P(A U B) = P(A) + P(B),** if A and B are mutually exclusive.

Random variable X – annual return on portfolio

Event A: return is less than -10%; P(A) = 10% Event B: return is more than 10%; P(B) = 20% P(AUB) = P(A) + P(B) = 30%

Note: Here A and B are exclusive because returns cannot be more than 10% and less than - 10% simultaneously.

### Exhaustive

All exhaustive events taken together form the complete sample space.

For the previous example, if we consider another event C: -10% ≤ return ≤ 10%

Now events A, B, and C are mutually exclusive and exhaustive. Thus P(A U B U C) = 1, i.e. P(C) = 70%

𝑃𝑃(𝐴𝐴) 𝑃𝑃(𝐵𝐵)

The probability that event A or event B would happen can be given as the sum of the three portions defined by the figure below:

𝑃𝑃(𝐴𝐴 ∩ 𝐵𝐵)

P(A) + P(B)−P(A ∩ B) P(A ∪ B) = �

P(A) + P(B) if A and B are Mutually Exclusive

𝑃𝑃(𝐴𝐴) 𝑃𝑃(𝐵𝐵)

𝑃𝑃(𝐴𝐴 ∩ 𝐵𝐵) = 0

**Joint Probability**

* + This is a statistical measure where the likelihood of two events occurring together and at the same point in time are calculated. In other words, *joint probability* is the probability of event Y occurring at the same time event X occurs.
	+ Notation for joint probability takes the form:
		- **P(X ∩Y) or P(X,Y)** Which reads the joint probability of X and Y.
	+ The following table shows the joint probability of different events. Let’s say an economist is predicting the market scenario and the price of IBM stock for the next year.
	+ Next year market can be Good, Bad, or Neutral.
	+ IBM stock may go up or go down.

* + Joint Probability Table:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Market** | **Good** | **Bad** | **Neutral** | **Total** |
| **IBM** |
| **UP** | 10% | 30% | 5% | *45%* |
| **DOWN** | 0% | 15% | 40% | *55%* |
| **Total** | ***10%*** | ***45%*** | ***45%*** | ***100%*** |

* + The probability of IBM stock being Up and the Market being Good is 10%.
	+ Similarly, the probability of IBM stock being down and the Market being neutral is 40%.

**Conditional Probability**

* + This is the probability of an event or outcome based on the occurrence of a previous event or outcome. Conditional probability is calculated by multiplying the probability of the preceding event by the updated probability of the succeeding event.
	+ The probability of event A given that event B has occurred is P(A/B), which is equal to the ratio of joint probability of A and B, and unconditional probability of B.

𝑃𝑃(𝐴𝐴 ∩ 𝐵𝐵)

𝑃𝑃(𝐴𝐴/𝐵𝐵) =

𝑃𝑃(𝐵𝐵)

The unconditional probability of market being Neutral is 45%. Then, by using the table below we can find 3 conditional probabilities.

* P(Up/Neutral) = 0.05/0.45
* P(Up/Good) = 0.1/0.1
* P(Down/Bad) = 0.15/0.45
* Joint Probability Table:

Unconditional Probabilities of IBM stock being UP/Down

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Market** | **Good** | **Bad** | **Neutral** | **Total** |
| **IBM** |
| **UP** | 10% | 30% | 5% | *45%* |
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Unconditional Probability

**Market**

45%

45%

10%

of Market

IMB Up

**Good**

Down Up

Down Up

Down

10% 10% 10% 10% 10% 10%

**Natural**

**Bad**

**Calculate:**

* Unconditional Probability of market to be good next year.
* Conditional Probability of IBM stock rising when the market is neutral.
* Conditional Probability of market being good when IBM stock is down.

A Random Variable is a function, which assigns unique numerical values to all possible outcomes of a random experiment under fixed conditions. A random variable is not a variable, is rather a function that maps events to numbers.

* Probability distribution describes the values and probabilities that a random event can take place. The values must cover all the possible outcomes of the event, while the total probabilities must sum to exactly 1, or 100%.

**Example**

* + Suppose you flip a coin twice.
	+ There are four possible outcomes: HH, HT, TH, and TT.

* + Let the variable X represent the number of Heads that result from this experiment
		- It can take on the values 0, 1, or 2.
		- X is a random variable (its value is determined by the outcome of a statistical experiment).
	+ A probability distribution is a table or a relation that links each outcome of a statistical experiment with its probability of occurrence.

|  |  |
| --- | --- |
| **Number of heads (X)** | **Probability P(X = x)** |
| 0 | 0.25 |
| 1 | 0.50 |
| 2 | 0.25 |

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| If a variable can take on any value between two specified values, it is called a continuous variable, otherwise, it is called a discrete variable. |

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| If a random variable is a discrete variable, its probability distribution is called a discrete probability.* For example, tossing of a coin and noting the number of heads (random variable) can take a discrete value.
* Binomial probability distribution, Poisson probability distribution.

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## (Cont.)

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| If a random variable is a continuous variable, its probability distribution is called a continuous probability distribution.* The probability that a continuous random variable will assume a particular value is zero.
* A continuous probability distribution cannot be expressed in tabular form.
* An equation or formula is used to describe a continuous probability distribution (called a Probability Density Function or Density Function or PDF).
* The area bounded by the curve of the density function and the x-axis is equal to 1 when computed over the domain of the variable.
* Normal probability distribution, and Student's t distribution are examples of continuous probability distributions.
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Thank You!

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