

**Valuations and Risk Models**

The term ‘Risk’ can be broadly defined as the degree of uncertainty about future net returns. Following are the related terms:



Credit risk relates to the potential loss due to the inability of a counterpart to meet its obligation.

Operational risk takes into account the errors that can be made in instructing payments or settling transactions.

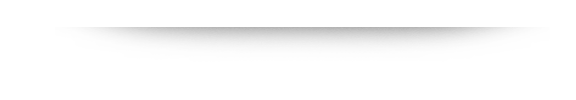
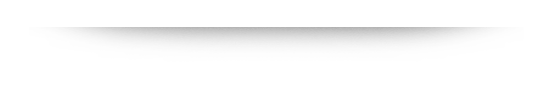
Liquidity risk is caused by an unexpected large and stressful negative cash flow over a short period.

Market risk estimates the uncertainty of future earnings, due to the changes in market conditions.

Broadly, the standard deviation of the variable measures the degree of risk inherent in the variable.

Say, the standard deviation of returns from the assets owned by you is 50%, and the standard deviation of returns from assets I own is 0%. We can say that risk of my assets is zero.

I own risk-less assets, as the standard deviation of returns



of my assets is 0%.

My assets are very risky, as the standard deviation of returns of my assets is 50%.

Traditional Mean – Variance Framework



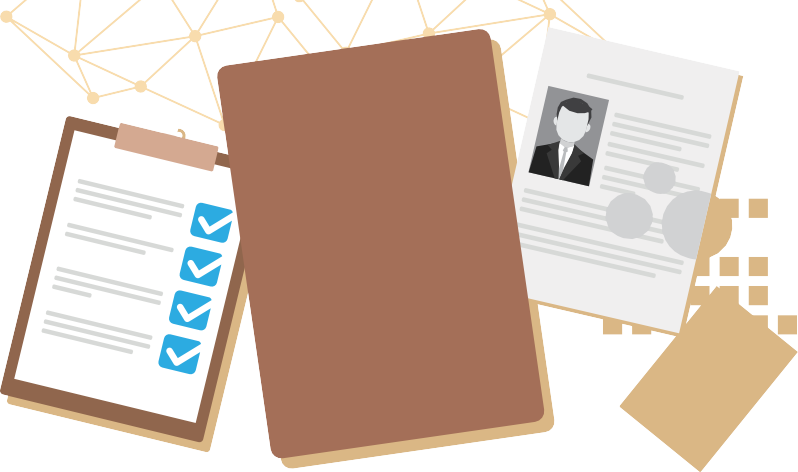
Return = Mean



Risk = Standard Deviation



Return Distribution = Elliptical Distribution (most common: Normal Distribution)



Mean can be calculated as: μP = w1μ1 + w2μ2

The Portfolio Mean with n investments can be calculated as: μP = � wiμi

i=1

n

ab

a b a b

2 2

b b

2 2

a a

w σ + w σ + 2w w ∗ σ ∗ σ ∗ ρ

σportfolio =

As the volatility of the portfolio can be calculated from the following expression:

The Portfolio Standard Deviation (Volatility) with n investments can be calculated as:

n n

σP = � � wiwjσiσjρij

i=1 j=1

## Minimum Variance Portfolios:



Mean, Standard Deviation, and correlation between different investment returns is consistent.



Mean and standard deviation only matters for a portfolio.



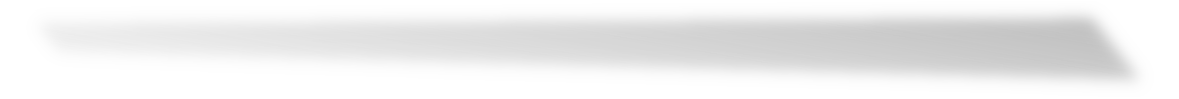
One can borrow at a risk-free rate.



Standard deviation is not the right measure for risk for non-normal distributions (non – symmetrical).



In real world, financial assets may have fatter tails and skew which defies the assumption of normal distribution.



Value at Risk (VaR) has become the standard measure that financial analysts use to quantify this risk.



VAR represents maximum potential loss in value of a portfolio of financial instruments with a given probability over a certain time horizon.



In simpler words, it indicates how much a financial institution can lose with probability (p) over a given time horizon (T).

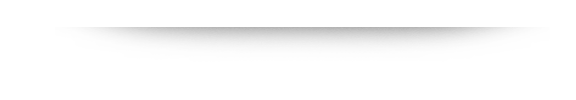


Say, the 95% daily VAR of your assets is $120, then it means that out of those 100 days there would be 95 days when your daily loss would be less than $120. This implies that during 5 days you may lose more than $120 daily.

There may be a day out of 100 when your loss is $5000,

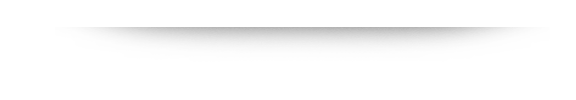
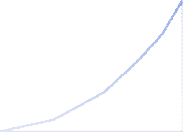
which means VAR does not tell anything about the extent to which we can lose.







* The colored area of the normal curve constitutes 5% of the total area under the curve.
* There is 5% probability that the losses will lie in the colored area, i.e., more than the VAR number.



0.45

Probability

0.4

0.35

0.3

0.25

95% daily-VAR

0.2

0.15

0.1

0.05

Z values

0

-4

-3

-2

-1

0

Mean = 0

1

2

3

4

|  |  |
| --- | --- |
| **Confidence (x%)** | **ZX%** |
| 90% | 1.28 |
| 95% | 1.65 |
| 97.5% | 1.96 |
| 99% | 2.32 |

0.45

0.4

0.35

0.3

0.25

0.2

0.15

0.1

0.05

0

-4

-2

0

Mean = 0

2

4

𝑉𝑉𝑉𝑉𝑅𝑅𝑋𝑋𝑋(𝑖𝑖𝑖𝑖𝑋) = 𝑍𝑍𝑋𝑋𝑋 ∗ 𝜎𝜎



ZX% : the normal distribution value for the given probability (x%) (normal distribution has mean as 0 and standard deviation as 1)



σ : standard deviation (volatility) of the asset (or portfolio)



VAR in absolute terms is given as the product of VAR in % and Asset Value:

𝑉𝑉𝑉𝑉𝑅𝑅 = 𝑉𝑉𝑉𝑉𝑅𝑅𝑋𝑋𝑋(𝑖𝑖𝑖𝑖𝑋) ∗ 𝑉𝑉𝐴𝐴𝐴𝐴𝐴𝐴𝐴𝐴𝑉𝑉𝐴𝐴𝐴𝐴𝐴𝐴𝐴𝐴



This can also be written as:

𝑉𝑉𝑉𝑉𝑅𝑅 = 𝑍𝑍𝑋𝑋𝑋 ∗ 𝜎𝜎 ∗ 𝑉𝑉𝐴𝐴𝐴𝐴𝐴𝐴𝐴𝐴𝑉𝑉𝐴𝐴𝐴𝐴𝐴𝐴𝐴𝐴



VAR for n days can be calculated from daily VAR as:

VaR(n days) (in 𝑋) = VaR(daily VaR) (in 𝑋) ∗ n



This comes from the known fact that the n-period volatility equals 1-period volatility multiplied by the square root of number of periods(n).

VaR(n days) (in 𝑋) = ZX𝑋 ∗ 𝜎𝜎∗Asset Value ∗ n



VaR for a portfolio:

VaRportfolio(in 𝑋)

=

𝑤𝑤 (𝑋VAR ) + w (𝑋VAR ) + 2w w ∗ (𝑋VAR )∗ (𝑋VAR )∗ 𝜌𝜌

a

2

a

2

2

2

b

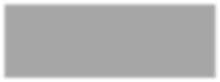
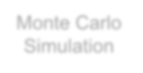
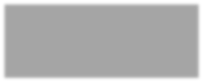
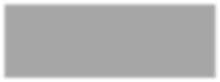
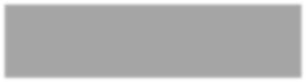
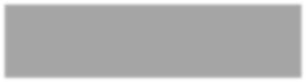
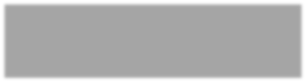
b

a b

a

b

ab



Full Revaluation Method

Linear Valuation Method

VaR

Monte Carlo Simulation

Historical Simulation

Delta Normal Method

Linear: When the value of the **delta is constant** for any change in the underlying –

* Primarily, in the case of **forwards and futures,** we have linear assets.
* The method to calculate VAR for linear assets is called Delta Normal method.
* The Delta Normal method assumes, that the variables are normally distributed.

Non Linear: When the value of the **delta keeps on changing** with the change in the underlying asset –

* Options are non-linear assets, where delta-normal method cannot be used as they assume the linear payoff of the assets.
* To calculate the VAR for non-linear assets, full revaluation of the portfolio needs to be done.
* Monte Carlo methods or Historical Simulation are commonly used to fully reevaluate the portfolio.

Payoff

linear

0

k

Share/asset price

Long position

Linear Derivatives: Payoff diagrams that are linear or almost linear:

* Forwards, futures

**Delta of Derivative: Change in price of Derivative to change in underlying asset.**

* Main reason for the difference is the shape of the payoff curve.



Option Price value

B

Slope = delta

A

Asset/Stock price

𝑓𝑓𝑓𝑓(𝑆𝑆0)(𝑆𝑆 − 𝑆𝑆0)2

𝑓𝑓(𝑆𝑆) = 𝑓𝑓(𝑆𝑆0) + 𝑓𝑓𝑓(𝑆𝑆0)(𝑆𝑆 − 𝑆𝑆0) + 2

𝑉𝑉𝐴𝐴𝑅𝑅 = 𝜇𝜇𝑃𝑃 + 𝑍𝑍𝜎𝜎𝑃𝑃

e

−(Z2)

2

ES = μ + σ

1 − x 2π

## For Delta Normal VAR:

* + A linear approximation is created.
  + The Approximation is an imperfect proxy for the portfolio.
  + It is computationally easy but may be less accurate.
  + The delta-normal approach (generally) does not work for portfolios of nonlinear securities.
  + E.g., Options VAR = Delta of Option \* (VaR at Zx%)
* Consider a portfolio of options dependent on a single stock price, S. Define:

Δ𝑆𝑆 =

Δ𝛿𝛿

𝛿𝛿

ΔP

Delta(𝛿𝛿) =

ΔS

* + Approximately:

Δ𝑃𝑃 = 𝛿𝛿Δ𝛿𝛿 = 𝛿𝛿𝛿𝛿Δ𝑆𝑆

* + For Many Underlying variables:

Δ𝑃𝑃 = � 𝛿𝛿𝐿𝐿 𝛿𝛿𝐿𝐿 Δ𝑆𝑆𝐿𝐿

𝐿𝐿

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Full Valuation method is the process of measurement of risk of a portfolio by fully re-pricing it under a set of scenarios over a time period. It can be used to cover a large range of values of the portfolio returns in order to provide more accurate results. It generally provides more accurate results compared to delta normal approach, but it is a complicated process.



Advantages over delta normal are as follows:

* + - It accounts for non-linearities of derivatives, whereas the delta normal assumes a linear approximation.
    - It accounts for extreme fluctuations.

Two popular methods under full revaluation approach have been explained in the subsequent slides.



Market conditions may cause the mean and variances to change over the period of time, which leads to fat-tailed distributions.



There are two explanations for the Fat Tails as follows:

* Conditional Mean: Mean changing over the period of time
* Conditional Volatility: Volatility changing over the period of time

4

2

0

-2

-4

95%

daily-VAR

0.45

0.4

0.35

0.3

0.25

0.2

0.15

0.1

0.05

0

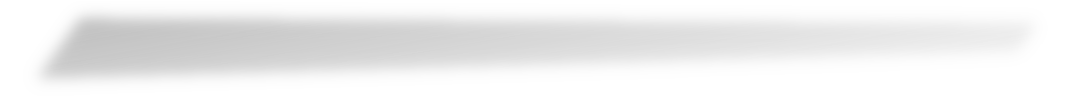




The fat-tailed unconditional distribution can be broken down into two conditional distributions, either with similar means and different variances or similar variances and different means.



Many a times, when we observe marked differences between the estimated and actual volatilities, it’s a result of regime switching which means that the average volatility in the market has now changed too much when compared to the previous estimate.



**Risk Management is all about understanding tails of distribution.**

* Historical based Approach:
  + Parametric Approach: It requires specific assumptions regarding the asset return distribution.
* Exponential smoothing
* Risk Metrics (EWMA model with λ = 0.94)
* The typical example of parametric approach is the delta-normal VAR.
  + Non-Parametric Approach: It is less restrictive, and it has no underlying assumptions of the asset return distribution.
* Historical Simulation: equal weighted returns
* Multivariate Density Estimation (MDE): analyze to get past periods corresponding to current period and then weight is assigned to historical data on how similar it is to current period.
* Implied Volatility based Approach



Thank You!

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